GCSE Maths - Algebra

Numerical Iteration (Higher Only)

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of numerical iteration questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Using a starting value of $x_0 = 9.3$, use numerical iteration to find the solution to the equation $x^2 - 10x + 6 = 0$ to 3 decimal places.

Step 1: Rearrange the equation so that it is in the correct form.

$$x^2 - 10x + 6 = 0$$
$$x^2 = 10x - 6$$
$$x = \sqrt{10x - 6}$$

Step 2: Add in the iteration notation.

$$x_{n+1} = \sqrt{10x_n - 6}$$

Step 3: Substitute in the starting value (x_0) for x_n to obtain x_1. Repeat until the same answer is obtained twice.

$$x_1 = \sqrt{10 \times 9.3 - 6} = 9.327 \ldots$$
$$x_2 = \sqrt{10 \times 9.327 - 6} = 9.342 \ldots$$
$$x_3 = \sqrt{10 \times 9.342 - 6} = 9.350 \ldots$$
$$x_4 = \sqrt{10 \times 9.350 - 6} = 9.354 \ldots$$
$$x_5 = \sqrt{10 \times 9.354 - 6} = 9.356 \ldots$$
$$x_6 = \sqrt{10 \times 9.356 - 6} = 9.357 \ldots$$
$$x_7 = \sqrt{10 \times 9.357 - 6} = 9.358 \ldots$$
$$x_8 = \sqrt{10 \times 9.358 - 6} = 9.358 \ldots$$

Now that we have the same answer twice (to 3 decimal places), this is our final solution.

$$\boldsymbol{x = 9.358}$$

Guided Example

Work out the solution to $x^3 - 15x + 12 = 0$ using numerical iteration, beginning with $x_0 = 3.3$. Give the solution to 3 decimal places.

Step 1: Rearrange the equation so that it is in the correct form.

$$x^3 - 15x + 12 = 0$$
$$x^3 = 15x - 12$$
$$x = \sqrt[3]{15x - 12}$$

Step 2: Add in the iteration notation.

$$x_{n+1} = \sqrt[3]{15x_n - 12}$$

Step 3: Substitute in the starting value (x_0) for x_n to obtain x_1. Repeat until the same answer is obtained twice.

$$x_1 = \sqrt[3]{15(3.3) - 12} = 3.3471 \ldots$$
$$x_2 = \sqrt[3]{15(3.3471) - 12} = 3.36808 \ldots$$
$$x_3 = \sqrt[3]{15(3.36808) - 12} = 3.37727 \ldots$$
$$x_4 = \sqrt[3]{15(3.377) - 12} = 3.3813 \ldots$$
$$x_5 = \sqrt[3]{15(3.3813) - 12} = 3.38306 \ldots$$
$$x_6 = \sqrt[3]{15(3.38306) - 12} = 3.38383 \ldots$$
$$x_7 = \sqrt[3]{15(3.38383) - 12} = 3.384167 \ldots$$
$$x_8 = \sqrt[3]{15(3.38417) - 12} = 3.38431 \ldots$$

stop since the answer is the same to 3 decimal place

The answer is: 3.384

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Calculate the solutions to the following, using numerical iteration. Give the solutions to 3 decimal places.

a) $x^2 + 3x - 80 = 0$, starting with $x_0 = 7.6$

① Rearrange the equation :

$x^2 + 3x - 80 = 0$ (+80 − 3x)

$x^2 = 80 - 3x$ (square root)

$x = \sqrt{80 - 3x}$

② Write the iteration notation :

$x_{n+1} = \sqrt{80 - 3x_n}$

③ Perform calculation :

$x_1 = \sqrt{80 - 3(7.6)} = 7.56306\ldots$

$x_2 = \sqrt{80 - 3(7.56306)} = 7.57038\ldots$

$x_3 = \sqrt{80 - 3(7.57038)} = 7.56893\ldots$

$x_4 = \sqrt{80 - 3(7.56893)} = 7.569226\ldots$

$x_5 = \sqrt{80 - 3(7.569226)} = 7.569169\ldots$

stop calculation because the same answer has been obtained to 3 decimal place.

The answer is 7.569

b) $2x^3 - 8x^2 - 5 = 0$, with a starting value of $x_0 = 4.1$

① Rearrange the equation :

$2x^3 - 8x^2 - 5 = 0$

$2x^3 = 8x^2 + 5$

$x^3 = 4x^2 + \frac{5}{2}$

$x = \sqrt[3]{4x^2 + \frac{5}{2}}$

② Write the iteration notation :

$x_{n+1} = \sqrt[3]{4(x_n)^2 + \frac{5}{2}}$

③ Perform calculation :

$x_1 = \sqrt[3]{4(4.1)^2 + \frac{5}{2}} = 4.116176\ldots$

$x_2 = \sqrt[3]{4(4.116176)^2 + \frac{5}{2}} = 4.1266\ldots$

$x_3 = \sqrt[3]{4(4.1266)^2 + \frac{5}{2}} = 4.1333\ldots$

$x_4 = \sqrt[3]{4(4.1333)^2 + 5/2} = 4.13766\ldots$

$x_5 = \sqrt[3]{4(4.13766)^2 + 5/2} = 4.14044\ldots$

$x_6 = \sqrt[3]{4(4.140)^2 + 5/2} = 4.14224\ldots$

$x_7 = \sqrt[3]{4(4.14224)^2 + 5/2} = 4.14339\ldots$

$x_8 = \sqrt[3]{4(4.1433)^2 + 5/2} = 4.14413\ldots$

$x_9 = \sqrt[3]{4(4.144)^2 + 5/2} = 4.14461\ldots$

$x_{10} = \sqrt[3]{4(4.1446)^2 + 5/2} = 4.14492\ldots$

} stop calculation

Answer: 4.145

c) $2x^3 + 4x = 14$, with a starting value of $x_0 = 1$

① Rearrange the equation:

$2x^3 + 4x = 14$

$-4x$: $2x^3 = 14 - 4x$

$\div 2$: $x^3 = 7 - 2x$

cube root: $x = \sqrt[3]{7 - 2x}$

② Write the iteration notation:

$x_{n+1} = \sqrt[3]{7 - 2x_n}$

③ Perform calculation:

$x_1 = \sqrt[3]{7 - 2(1)} = 1.7099...$

$x_2 = \sqrt[3]{7 - 2(1.7099)} = 1.5298...$

$x_3 = \sqrt[3]{7 - 2(1.5298)} = 1.57948...$

$x_4 = \sqrt[3]{7 - 2(1.5794)} = 1.56609...$

$x_5 = \sqrt[3]{7 - 2(1.56609)} = 1.5697...$

$x_6 = \sqrt[3]{7 - 2(1.5697)} = 1.5687...$

$x_7 = \sqrt[3]{7 - 2(1.5687)} = 1.5690...$

$x_8 = \sqrt[3]{7 - 2(1.569)} = 1.5689...$

stop calculation

Answer: 1.569

d) $0.5x^3 + 2.5x - 10 = 0$, with a starting value of $x_0 = 2$

① Rearrange the equation:

$0.5x^3 + 2.5x - 10 = 0$

$-2.5x$, $+10$: $0.5x^3 = 10 - 2.5x$

$\div 0.5$: $x^3 = 20 - 5x$

cube root: $x = \sqrt[3]{20 - 5x}$

② Write the iteration notation:

$x_{n+1} = \sqrt[3]{20 - 5x_n}$

③ Perform calculation:

$x_1 = \sqrt[3]{20 - 5(2)} = 2.15443...$

$x_2 = \sqrt[3]{20 - 5(2.154)} = 2.09749...$

$x_3 = \sqrt[3]{20 - 5(2.09749)} = 2.1189...$

$x_4 = \sqrt[3]{20 - 5(2.1189)} = 2.11089...$

$x_5 = \sqrt[3]{20 - 5(2.11089)} = 2.11386...$

$x_6 = \sqrt[3]{20 - 5(2.11386)} = 2.11274...$

$x_7 = \sqrt[3]{20 - 5(2.11274)} = 2.11316...$

$x_8 = \sqrt[3]{20 - 5(2.11316)} = 2.11300...$

stop calculation

Answer: 2.113